Multiobjective Programming for Generating Alternatives: A Multiple-Use Planning Example

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ABSTRACT. This paper describes two related issues pertaining to the role of multiple objective programming (MOP) in complex forest planning problems. First, some implications of using inadequate MOP models in complex forest planning problems are described. Second, an alternative MOP approach—modeling to generate alternatives—is presented and illustrated using a multiple-use planning example. For. Sci. 33(2):458–468.

ADDITIONAL KEY WORDS. Forest planning, linear programming, goal programming, nondominated solutions, incomplete models.

MULTIPLE OBJECTIVE PROGRAMMING (MOP) encompasses a general class of mathematical programming techniques for solving problems in which several objectives are considered simultaneously. Multiple-use forest planning exemplifies this situation because it involves several objectives or goals such as increased revenues from timber resources, improved water quality, protection of wildlife, and increased recreational opportunities.

The use of mathematical programming techniques in multiple-use forest planning has been limited mainly to linear programming (Leuschner et al. 1975, Kent 1980, Bare et al. 1984, Johnson 1986), and especially goal programming (Field 1973, Bell 1976, Rustagi 1976, Schuler et al. 1977, Field et al. 1980, Hotvedt et al. 1982, Arp and Lavigne 1982, Walker 1985). Other MOP techniques have also been proposed (Steuer and Schuler 1978, Bertier and deMontgolfier 1974, Bare and Mendoza 1986, Allen 1986, Hof et al. 1986, Harrison and Rosenthal 1986).

Most MOP techniques are designed to generate, identify, or select non-dominated solutions. Intuitively, the selection of a nondominated solution is appealing because, from the standpoint of rational decision-making, no other solution leads to better attainment of the stated objectives. This rationale is valid and applies in situations where the planning problem may be considered relatively simple and well defined so that important issues, concerns, and objectives can be adequately "modeled" or included in the formulation and design of the model.

Some planning problems, however, are complex or wicked with broad and ill-defined boundaries (Liebman 1976). Hence it is conceivable that not all aspects of the problem can be adequately captured within a mathematical

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A feasible solution is nondominated if there exists no other solution that will yield an improvement in one objective without causing a degradation in at least one other objective.

programming framework (Allen and Gould 1986). Some issues or concerns may be omitted because they are inherently qualitative in nature or because inadequate scientific information precludes their incorporation. Moreover, other objectives may be hidden, unrevealed, or possibly unknown to the decision-maker or analyst (Brill 1979). Hence the "formulated" MOP model may be considered inadequate in capturing all of the quantitative and qualitative elements of the planning problem.

This paper examines the use of MOP in complex forest planning problems. In addressing this topic, two separate yet related issues are discussed. First, a simplified two-objective problem is presented to show that under complex decision environments where the formulated MOP model may be inadequate, a preferred solution (in terms of the complete or "true" model) may be dominated if considered only in the context of the inadequate or formulated model. This leads to the second issue, which suggests that to mitigate against this possibility, a general philosophy different from most of the existing MOP techniques must be adopted. The general thesis of this approach—modeling to generate alternatives (MGA)—is that the decision-maker should be presented with a variety of solutions that are maximally different in terms of the decision variables, yet satisfactory with respect to the modeled objectives. By examining a wide array of diverse solutions, it is presumed that the decision-maker will be in a better position to make a final choice vis à vis the modeled objectives and in so doing satisfy objectives omitted because they were too difficult to explicitly model or were initially hidden or unknown to the decision maker.

A Simplified Two-Objective Problem

For illustrative purposes, consider the simple two-objective problem shown in Figure 1. In this example, the complete or true MOP model includes both objectives 1 and 2, while the inadequate or formulated model is assumed to include only objective 2. Suppose only objective 2 is maximized producing

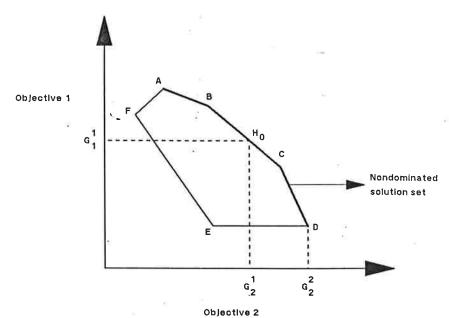


FIGURE 1. Feasible region in objective space.

solution D with maximum value equal to G_2^2 . Next, suppose the first objective is also considered and is simultaneously optimized with objective 2 (i.e., using the complete or true MOP model). Using any one of several MOP techniques (see Bare and Mendoza 1986, Cohon 1978, Hwang and Masud 1979, Zeleny 1982) to solve the complete model, a compromise solution along the lines ABCD will be generated. For instance, if goal programming is used, and the target levels for objectives 1 and 2 are set at G_1^1 and G_2^1 , respectively, the solution H_O is generated. Other solutions along ABCD may be generated by goal programming given different sets of goal levels and weights (or priorities) assigned to each goal (Ignizio 1982). Clearly, the compromise solution (H_O) for the complete or true model would not be selected if only nondominated solutions from the inadequate or formulated model are considered.

The implication of the simple illustration is that if an inadequate or formulated MOP model is used in the analysis, seemingly inferior solutions should not be automatically excluded in the search for the preferred compromise solution. This is because preferred solutions (from the standpoint of the complete or true model) may be dominated in the context of the inadequately formulated MOP model.

It may appear from the illustration that added information should be provided before a rational decision (e.g., H_O in Figure 1) can be made. Further, one may argue that if decision-makers use the formulated model, then in the absence of information about the "omitted" objective(s), how can the choice of a dominated solution (H_O) be justified. Obviously, such a choice cannot be justified, even if it is the correct decision. In fact, even the true model may prove to be inadequate when compared to the real world system it purports to represent. This follows the well-known observation that all models are abstractions of reality, and hence are subject to formulation errors. To help mitigate against the consequences of adopting solutions from inadequately formulated models, MGA is suggested as an alternative approach.

Modeling to Generate Alternatives

In the context of the above observations, a different view about the role of MOP in complex forest planning problems is proposed. Instead of using MOP models to "optimize" (i.e., identify and choose nondominated solutions), we believe they should be viewed as tools that can be used to generate "satisfactory" alternative solutions. Hence, the emphasis when designing and using MOP models should not be on optimization per se, but on the generation of "satisfactory" alternatives (Liebman 1976). Following this view, Brill (1979) advocates the MGA approach.

MGA is a general approach for generating alternative solutions that are (1) satisfactory with respect to the objectives (i.e., objective space), and (2) maximally different with respect to the decisions (i.e., decision space). The first criterion can be met if the formulation specifies minimum requirements, including target levels for each objective, as constraints. The second criterion requires that the generated solutions be maximally different in terms of the decision variables. Hence, MGA can be used as a tool in generating solutions that are "satisfactory" with respect to the known or "modeled" objectives, and widely different with respect to the decision variables.

The primary motivation of this approach is that by generating a sufficient number of distinctly different and satisfactory alternatives, a wider span of the decision space can be searched, providing a wider range of choices for the decision-maker to consider. Implicitly, this approach assumes that by providing a wider range of distinctly different solutions, the decision-maker will be in a better position to select the preferred solution. Moreover, the possible impact of the missing or unmodeled objectives may be implicitly considered within the wider set of generated solutions. It is possible that with the opportunity to examine a wide range of distinct solutions, some insights may be gained, particularly on issues and concerns that may not have been initially considered in the model, leading to a more rational deci-

The use of models to generate alternative solutions in MOP is an approach that, in a strict sense, may not be unique to MGA. Other models also generate alternative solutions mainly by manipulating the model parameters through sensitivity analysis. For instance, Hotvedt et al. (1982), and Field et al. (1980), describe how various alternatives can be generated through goal programming by manipulating the goal levels and weights (or priorities) of each objective. The approach of Hotvedt et al. (1982) in some ways parallels MGA because it also generates alternative solutions using a heuristic weight determination procedure. However, the generation of alternative solutions in Hotvedt et al. (1982) is primarily through manipulation of the model parameters, particularly the weights. Hence, its emphasis is primarily in the objective space, not in the decision space as is true in MGA.

In contrast, MGA is an approach that aims to generate maximally different solutions in terms of the decision variables. MGA does not generate alternative solutions through a systematic manipulation of the model parameters. Instead, MGA concentrates on the variability of the alternative solutions with respect to decision space. As stated earlier, while there may be numerous satisfactory solutions with respect to the known or modeled objectives, they may be significantly different from each other with respect to the decisions they specify. And, some of these solutions may be better than others with respect to the unknown, unmodeled, or hidden issues and con-

One of the methods utilizing this approach is the Hop, Skip, and Jump (HSJ) method (Brill et al. 1982). The general form of this method is discussed below:

Step 1. Obtain an initial solution using any method (e.g., maximize one of the objectives or use goal programming).

Step 2. Obtain an alternative solution by solving

$$Min = \sum_{j=1}^{J} X_{j}$$
s.t. $Z_{k}(X) \ge T_{k}$; $k = 1, 2, ..., K$ (2)

s.t.
$$Z_k(X) \ge T_k$$
; $k = 1, 2, ..., K$ (2)

$$x \in X$$
 (3)

where

J = set of indices of the decision variables that are nonzero (i.e., basic)in the original solution,

 T_k = target specified for objective $Z_k(X)$ $x \in X$ = set of feasible solutions

 $X_i = \text{basic variable } j$

The formulation above is designed to produce alternative solutions different from those previously generated. Note that the nature of the objective function in (1) suggests that the algorithm will generate a new set of basic variables (i.e., a new set of decision variables and hence a distinct new solution) by driving some of the previously basic variables to zero. In the extreme case, the formulation drives all previous nonzero decision variables to zero, resulting in an alternative set of decision variables completely different from the previous solutions. In a less extreme case, only some of the previously nonzero decision variables are driven to zero, resulting in a "less distinct" alternative set of nonzero decision variables. However, the target levels T_k ensure that the new solution generated will be satisfactory with respect to the modeled objectives.

Step 3. Generate a third solution different from the first two by solving a problem analogous to Step 2. Generate a series of additional alternative solutions following the same process.

A Multiple-Use Planning Example

The forest planning problem considered in this paper is adapted from Bell (1976) and is used solely to demonstrate the method. The problem involves the allocation of forest lands within homogenous management units to various land uses. Five objectives are considered: (1) maximization of timber yield, (2) maximization of forage production, (3) increased developed recreation, (4) improved dispersed recreation, and (5) water production. There are three management units with a total area of 45,000 ac. The annual output matrix for the three management units is described in Table 1. Table 2 is a pay-off table containing the maximum values of the five objectives when they are optimized independently, including the respective values of the other objectives. Clearly, the "ideal" solution wherein all five objectives are simultaneously optimized is not feasible.

For demonstrative purposes, assume that one of the objectives, particularly dispersed recreation, is omitted in the MOP formulation. Hence, the inadequately "formulated" MOP model includes only the remaining four objectives. The results of this partial MOP analysis will be examined later relative to the missing or omitted dispersed recreation objective.

Following the steps of the algorithm as described earlier, the solution that maximizes the timber objective is arbitrarily chosen as the initial alternative in Step 1. Per Table 3, or solution 1 in Table 2, this alternative yields 6,650 mbf of timber; 8,125,000 lb of forage; 27,000 ac-ft of water, and no visitor-days of developed recreation.

For Step 2 of the algorithm, the following target levels are used: (1) 6,000 mbf of timber, (2) 8,000,000 lb of forage, (3) 10 million visitor-days of developed recreation, and (4) 20,000 ac-ft of water. These values are less than the individual maximum values of each objective, but are assumed to be satisfactory. The basic variables (i.e., nonzero decision variables) from the initial solution in Step 1 are identified and included in (1). The solution generated in Step 2 is denoted by HSJ1 in Table 3. Again, the basic variables from the HSJ1 solution are identified and included in (1) to generate the next solution, denoted by HSJ2. The process is repeated as other alternatives are generated. A summary of the HSJ solutions generated for the sample problem are presented in Tables 3 and 4.

In general, the stopping criteria for the HSJ procedure are (Brill 1982): (1) when no new basic variable can enter the solution [i.e., can be included in (1)] because all decision variables are currently or have previously been included in the HSJ objective function described in (1), and (2) when the analyst or decision-maker feels that a sufficient number of alternatives have

TABLE 1. Planning unit annual output and constraint matrix for three management units.

		Mans	Management Unit A	Unit A		Mana	Management Unit B	nit B		Manag	Management Unit C	nit C		
					Manag	gement a	Management activity by primary objective	primary	objective	4)				
Output	Dispersed recreation	Forage	Wilderness	No management	Developed recreation	Maximum 19dmit	Dispersed recreation	Богаде	Maximum timber	Developed recreation	Modified timber management	Dispersed recreation	Forage	=
							Units/a		Units/ac					
Objectives		ÿ.		¥1		6						5	2	*
 Timber (mbt) Forage (lb) 	100	150	100	100	30	0.25	0.22	0.24	0.15 250		0.1 245	0.12 250	275	
3. Developed recreation										;			•00	
(visitor-days)					2,000	ū.				1,500				
4. Dispersed recreation (visitor-days)	~	0.5	0	0		0.5	0.75	0.5	S		9	5.5	0.5	
5. Water (ac-ft)	0.8	0.8	0.8	0.8	6.0	1.0	1.0	1.0	0.2	0.2	0.2	0.2	0.2	
Constraints														
6. Sediment (tons)	4	4	ω ·	3	4	5	4.5	S	7	7	9	9	9	≤235,000
 Maximum wilderness (ac) Minimum wilderness (ac) 														≅4,000 ≥2,000
9. A (ac)	-	-	1	-	-									= 10,000
10. B (ac)						-		-	200	_	-	ù .	_	= 15,000 = 20.000
12. Recreation zone (ac)										-300	-	8		0

TABLE 2. Maximum and attainable values of each objective when maximized separately.

		Obje			
Solutions	Timber (mbf)	Forage ('000) (lb)	Dev. rec. ('000) (vd)	Water (ac-ft)	Dis. rec (vd)
1	6,650	8,125	0	27,000	63,500
2	5,600	9,000	0 .	27,000	21,700
3	5,743	7,198	16,099	27,800	17,667
4	6,150	7,315	16,000	33,800	117,700
5	5,293	7,198	9,967	27,000	147,051

been generated, or when the difference between each new alternative and a previous one becomes insignificant. The first stopping criterion suggests that all decision variables have been basic at some previous iteration, and hence they have been included in the objective function at one time or another. Some variables may consistently be nonzero and will consistently be in the basis. Others may, at some iteration, be nonzero and then become zero at a succeeding iteration. If the first criterion is imposed, it implies that all decision variables have exhaustively been considered and have at some iteration been part of a generated alternative solution.

The second criterion is more general and more flexible. Its use implies that two alternative solutions that are successively generated are not very different in terms of the decision variables that are nonzero. Or, the decision-maker may decide that enough solutions have been examined.

The HSJ algorithm is very flexible and lacks the rigid structure and systematic procedural search of other algorithms. For instance, the choice of the initial solution is completely arbitrary, and hence the algorithm can be restarted at any time using a different initial solution. Obviously, if the algorithm is intended to be used as an optimization tool, it would be inefficient. However, if the algorithm is used to generate maximally different but satisfactory alternatives, it is flexible enough to generate a sufficiently wide range of distinct solutions.

Differences Among Generated Solutions

As pointed out earlier, the HSJ approach is designed to generate alternative solutions that are (1) satisfactory relative to the targets specified in (2), and (2) distinctly different from each other in decision space. This section describes the differences among the four solutions generated by HSJ.

Table 3 shows the solutions in objective space (i.e., values of the objec-

TABLE 3. Summary of HSJ solutions in terms of the values of the objectives.

	Timber (mbf)	Forage ('000 lb)	Dev. rec. ('000 vd)	Water (ac-ft)	Missing objective (Disp. rec.) (vd)
Initial	6,650	8,125	0	27,000	63,500
HSJ1	6,080	8,000	10,000	27,500	99,200
HSJ2	6,100	8,000	10,000	27,500	25,950
HSJ3	6,000	8,000	10,000	27,500	79,426
HSJ4	6,000	8,000	11,623	27,579	73,157

TABLE 4. Land use allocation for various HSJ solutions.

		No., of new basic variables	5	4	7	1	0
		·· Forage	10,000	4,000	20,000	9,013	10,082
Unit C		Dispersed noitsercen		16,000			
Management Unit C		Modified timber management				10,951	9,885
Mar		Developed recreation				36	33
	ective	Maximum timber	10,000				
nit B	rimary obj	эдвло П		15,000		10,685	
Management Unit B	Management activity by primary objective	Dispersed recreation			15,000		15,000
Mana	agement ac	Maximum 19dmit	15,000			4,315	
	Mana	Developed Tecrestion		5,000	5,000	4,973	5,787
nit A		No management	8,000				
nagement Unit A		Wilderness	2,000	2,000	2,000	2,000	2,000
Mana		ЭдвтоЧ		3,000	1,000	3,027	2,213
		Dispersed recreation			2.000		
		Solution	Initial	HS11	HSJ2	HSJ3	HSJ4

tives) while Table 4 contains the solutions in decision space (i.e., acres within a management unit managed under a given treatment alternative). Looking at Table 4, the differences among the HSJ solutions can be expressed in terms of (1) the number of new land uses, and (2) changes in the acreages of land uses. The differences between alternatives in objective space as shown in Table 3 can be expressed in terms of (1) changes in the values of the objectives and (2) changes in the values of the missing objective (i.e., dispersed recreation).

DIFFERENCES IN THE NUMBER OF LAND USES

The column in Table 4 represented by the number of new basic variables corresponds to the number of new land uses that were not previously generated. For example, consider the initial and HSJ1 solutions. The initial solution has five land uses, namely (1) 2,000 ac of wilderness in management unit A, (2) 8,000 ac of no management (idle) lands in A, (3) 15,000 ac of timberland in B, (4) 10,000 ac under timber management in C, and (5) 10,000 ac of forage in C. Looking at the solution in HSJ1, there are four new land uses that were not generated in the initial solution, namely 3,000 ac of forage in A, (2) 5,000 ac of developed recreation in A, (3) 15,000 ac of forage in B, and (4) 16,000 ac of dispersed recreation in C. Hence, the initial solution and the HSJ1 solution are different in two ways: (1) there are four new land uses in HSJ1 that were absent in the initial solution, and (2) three land uses in the initial solution are dropped in the HSJ1 solution. Hence, the two solutions offer two different sets of land uses with their corresponding outputs as shown in Table 3.

DIFFERENCES IN ACREAGES OF LAND USES

Table 4 shows the differences among HSJ solutions in terms of the changes in the values (acreages) of each land use. For instance, the allocation for forage in management unit C varies from 4,000 ac in HSJ1 to 20,000 in HSJ2. Or, timber management in B varies from 15,000 ac for the initial solution to 4,315 ac in HSJ3 and no allocation in HSJ1, HSJ2, and HSJ4.

CHANGES IN THE VALUES OF THE OBJECTIVES

The difference in the HSJ solutions can also be observed in objective space as shown in Table 3. For instance, water production ranges from 27,000 ac-ft for the initial solution to 27,579 ac-ft in HSJ4. Also, the values of the developed-recreation objective range from 10 million to 11.623 million visitor-days, while the timber objective ranges from 6,000 to 6,650 mbf. With the exception of the missing objective (dispersed recreation) the different solutions do not appear to yield significantly different outputs. This result again illustrates the possibility that some alternatives may be nearly as good as each other with respect to the modeled objectives, while others may be better with respect to the unmodeled objectives. This observation is further described in the next section.

DIFFERENCES IN TERMS OF THE MISSING OBJECTIVE

One of the issues raised and discussed in this paper is the implication of choosing solutions generated from an inadequately formulated MOP model. To illustrate, we have omitted the dispersed recreation objective in the HSJ analysis. From Table 3, it is clear that the values of the dispersed recreation objective vary considerably among the HSJ solutions. The values range from 25,950 visitor-days in solution HSJ2, to a maximum of 99,200 visitor-days in the HSJ1 solution. This implies that although the values of

the modeled objectives do not vary significantly among the solutions of the inadequately formulated model, these solutions may in fact be very distinct relative to the complete or true model.

Another implication that can be observed from Table 3 is described in solutions HSJ1 and HSJ2. Note that considering only the first four objectives, HSJ1 is dominated by HSJ2 because HSJ2 yields more timber volume while producing the same amount of forage, developed recreation, and water. However, the results based on the true five objective MOP model show that when the missing objective is included, HSJ1 is no longer dominated. Again, this illustrates our contention that because of possible formulation omissions, solutions need not be categorized as dominated and non-dominated. Consequently, seemingly dominated solutions from an 'inadequately' formulated model (like the four-objective multiple-use planning example), should not be categorically disregarded. And, to guard against this occurring, the MGA approach is suggested as a possible way to further explore alternative solutions that produce satisfactory results.

Summary and Conclusions

MOP models formulated for analyzing complex forest planning problems may be inadequate because relevant concerns, issues, or objectives may be inadequately included or inadvertently omitted from the model. When this inadequately formulated model is used in the analysis, dominated solutions should not be excluded in the search for the preferred alternative because potentially they may be better alternatives.

This paper has described an alternative approach that may be better suited for this type of problem. The primary emphasis of the approach is the generation of maximally distinct and satisfactory solutions, rather than the identification of an optimal or nondominated solution. One method utilizing this general approach, called Hop, Skip, and Jump is described using a multiple-use planning example.

The problem of missing objectives or an inadequately formulated model in complex planning problems is a realistic one. The approach described in this paper recognizes this problem and considers it in an implicit manner. While the proposed approach may not be superior to other MOP techniques, it offers an alternative approach especially in complex forest planning situations. One apparent disadvantage of the approach is the lack of a systematic procedure for aiding the decision-maker in selecting the best solution from among the set of generated solutions. It is, however, plausible that this approach may prove to be better suited to decision-making under complex or wicked environments such as forest planning. Further, this is also a disadvantage of other MOP techniques that modify problem parameters through sensitivity analysis.

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